International Journal of Thermophysics, Vol. 15, No. 3, 1994

Simultaneous Measurements of Specific Heat and Total Hemispherical Emissivity of Chromel and Alumel by a Transient Calorimetric Technique

S. Sasaki,[1] H. Masuda,[2] M. Higano,[2] and N. Hishinuma[2]

Received October 25, 1993

Using a transient calorimetric technique, the specific heat and total hemispherical emissivity of chromel and alumel were measured simultaneously in the temperature range 360–760 K. Two types of specimens for each material were prepared. To obtain reliable experimental values of specific heat and total hemispherical emissivity, an expression for the time history of the temperature of the specimens was developed; this expression is accurate over the whole temperature range. An error analysis is made and the uncertainty (the total error) in the values of specific heat and total hemispherical emissivity is estimated to be 3.1% for the well-designed specimens.

KEY WORDS: emissivity; heat transfer; metal; specific heat; thermal radiation.

1. INTRODUCTION

Data on the thermophysical properties of metallic substances are very important for the development of new and advanced engineering materials. In response to this need, techniques for the precise, multiple measurement of thermophysical properties, including emissivity, have been developed, for example, by Cezairliyan [1, 2], by Taylor [3], and more recently, by Takahashi and Sugawara [4]. Although it has many advantages, the resistive self-heating method may not always give good results, especially at temperatures below 900 K [4]. A new measurement method which has been reported in a previous paper [5], using the transient calorimetric technique [6], permits the total hemispherical emissivity and specific heat

---

[1] Department of Mechanical Engineering, Ichinoseki National College of Technology, Hagisho, Ichinoseki 021, Japan.

[2] Institute of Fluid Science, Tohoku University, Katahira, Aoba-ku, Sendai 980, Japan.

0195-928X/94/0500-0547$07.00/0 © 1994 Plenum Publishing Corporation

of a metal to be measured simultaneously. This simultaneous-measurement method is useful, particularly for cases where the specific heat of the sample metal is unknown.

The purpose of this study is twofold: (i) to obtain data on the specific heat and total hemispherical emissivity of chromel and alumel for accurate evaluation of the heat losses through chromel and alumel leads when used as a thermocouple, and (ii) to ascertain the uncertainty (total error) in the specific heat and emissivity values obtained by the present measurement method.

2. EXPERIMENTS

In this study, chromel and alumel, commonly used as thermocouple materials, were selected as the materials whose properties were to be measured. The chemical compositions (% by weight) are 91.2 Ni, 7.88 Cr, 0.71 Mn, 0.52 Si, 0.12 Al, 0.037 Fe, and 0.002 Mg for chromel and 95.5 Ni, 1.92 Si, 1.29 Mn, 0.99 Al, 0.33 Fe, 0.017 Cr, and 0.003 Mg for alumel. The principle of the simultaneous measurement of the specific heat c and total hemispherical emissivity ε_h of a metal is described in detail in a previous paper [4], and only a brief description is given here. For the simultaneous measurement, a set of two specimens is needed: Specimen I is a specimen made of the metal whose thermophysical properties have to be measured (chromel or alumel in this work; referred to hereafter as metal A), and specimen II is a compound specimen composed of metal A and a standard reference metal (referred to as metal B). Copper (99.9% purity) was used as metal B. Following the work of Masuda et al. [5] and Masuda and Higano [6], the configurations used for specimens I and II were a disk and a concentric-cylinder shape, respectively. Both specimens are shown in Fig. 1; D is the diameter and L is the thickness of the specimens. Moreover, for specimen II, two specimens were prepared: Specimen II-1, a hollow-cylinder shape of metal A, $(D_a - D_b)/2$, is thin; and specimen II-2 is thick, where D_a is the outer diameter of metal A and D_b is the diameter of metal B. Specimen II-1 is preferable to specimen II-2, satisfying Eq. (21), given later. The specifications for all the specimens used are given in Table I. To all specimens, one or two thermocouples were attached for measuring the specimen temperatures, as shown in Fig. 1. Specimen II-1 was suspended by a thermocouple attached to metal B, while specimen II-2 was suspended by two thermocouples attached to metals A and B. The reason for this difference is mentioned in Section 4.1. The thermocouples used here were iron–constantan, with diameters of 25 and 50 μm for specimens I and II, respectively. The specimens were mechanically polished and washed

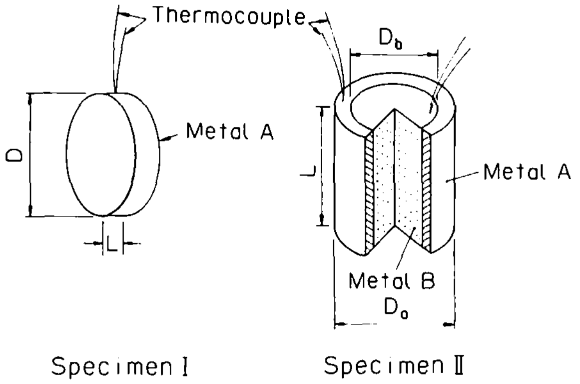

Fig. 1. Shapes and configurations of specimens I and II.

thoroughly in distilled water, similar to those used in the previous papers [5, 6].

An improved experimental apparatus was used in this study in place of the apparatus employed in the previous work [5]. The new apparatus is shown schematically in Fig. 2. The main improvement in the present apparatus was the installation of the water-cooled radiation shield cover (4) in the vacuum chamber (1). The radiation from the electric furnace (8)

Table I. Physical Description of the Specimens[a]

Chromel specimens
- Specimen I (chromel)
  - $D = 8.20$, $L = 1.46$, $m_{\mathrm{chr}} = 0.653$
- Specimen II-1 (chromel and copper)
  - $D_a = 8.31$, $D_b = 7.33$, $L = 19.93$, $m_{\mathrm{chr}} = 2.005$, $m_{\mathrm{cop}} = 7.524$
- Specimen II-2 (chromel and copper)
  - $D_a = 8.07$, $D_b = 5.16$, $L = 20.00$, $m_{\mathrm{chr}} = 5.130$, $m_{\mathrm{cop}} = 3.752$

Alumel specimens
- Specimen I (alumel)
  - $D = 8.17$, $L = 1.45$, $m_{\mathrm{alu}} = 0.638$
- Specimen II-1 (alumel and copper)
  - $D_a = 9.29$, $D_b = 8.26$, $L = 19.91$, $m_{\mathrm{alu}} = 2.359$, $m_{\mathrm{cop}} = 9.565$
- Specimen II-2 (alumel and copper)
  - $D_a = 8.48$, $D_b = 4.95$, $L = 19.95$, $m_{\mathrm{alu}} = 6.308$, $m_{\mathrm{cop}} = 3.440$

[a] Units: D in mm, L in mm, and m in g. Subscripts: chr, chromel; alu, alumel; and cop, copper.

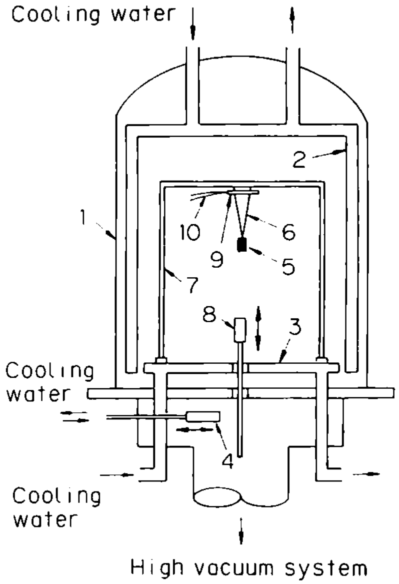

Fig. 2. Schematic drawing of the experimental apparatus: (1) vacuum chamber; (2) upper cooled bath; (3) lower cooled bath; (4) radiation shield cover; (5) specimen; (6) thermocouple leads suspending specimen; (7) support; (8) electric furnace for heating specimen; (9) terminal box with guard heater; (10) thermocouple leads for terminal box.

could be intercepted by the movable shield cover, after heating of the specimen was completed and the furnace was moved downward. Then radiative cooling of the specimen (5) was begun, and the temperature decay was recorded digitally. The other parts of the apparatus were nearly the same as described in the previous paper [5]. Therefore, their descriptions are omitted here.

The upper and lower baths (2, 3) were cooled by water, and their wall temperatures were kept at about 294 K during the measurements. The measurements of specimen temperatures were performed in the range of 360–760 K; the vacuum was better than 4×10^{-4} Pa.

3. DATA ANALYSIS FOR DETERMINATION OF SPECIFIC HEAT AND TOTAL HEMISPHERICAL EMISSIVITY

After the cooling rates of specimens I and II, $dT_{s,1}/dt$ and $dT_{s,2}/dt$, have been determined, respectively, from the corresponding cooling curves, i.e., the $T_{s,1}$ versus t and $T_{s,2}$ versus t curves, the specific heat c_a and the total hemispherical emissivity $\varepsilon_{h,a}$ of metal A can be obtained using the cooling rates from the following equation [4]:

$$c_a(T_s) = \frac{\begin{bmatrix} m_{b,2} c_b(T_s)(dT_{s,2}/dt)|_{T_s} \\ \quad + \sigma F_{b,2}[\varepsilon_{h,b}(T_s)\, T_s^4 - \alpha_{h,b}(T_s)\, T_\infty^4] - (F_{a,2}/F_{a,1})\, Q_{t,1} + Q_{t,2} \end{bmatrix}}{m_{a,1}[(F_{a,2}/F_{a,1})(dT_{s,1}/dt)|_{T_s}] - m_{a,2}(dT_{s,2}/dt)|_{T_s}} \tag{1}$$

$$\varepsilon_{h,a}(T_s) = \frac{-m_{a,1} c_a(T_s)(dT_{s,1}/dt)|_{T_s} + \sigma \alpha_{h,a}(T_s)\, F_{a,1}\, T_\infty^4 - Q_{t,1}}{\sigma F_{a,1}\, T_s^4} \tag{2}$$

where m, F, and T_s are the mass, surface area, and temperature of the specimen, respectively. T_∞ is the surrounding wall temperature, t is time, σ is the Stefan–Boltzmann constant, and α_h is the total hemispherical absorptivity of the specimen. Q_t is the conductive heat loss through thermocouple leads suspending the specimen. Subscripts a and b refer to metals A and B, and subscripts 1 and 2 refer to specimens I and II, respectively. It should be noted that the specific heat $c_a(T_s)$ and the emissivity $\varepsilon_{h,a}(T_s)$ can be obtained from Eqs. (1) and (2), respectively, when the temperatures of specimens I and II are equal to each other, that is, $T_{s,1} = T_{s,2} = T_s$.

The absorptivities of the specimen, $\alpha_{h,a}$ and $\alpha_{h,b}$, appear in Eqs. (1) and (2). These equations differ from the corresponding equations in the work of Masuda et al. [5], because the cooled baths in the present apparatus are cooled by water so that T_∞ is comparatively high and the approximation $\alpha_h = \varepsilon_h$ can no longer be applied. In the present work the following approximation, made by Eckert [7], is used for convenience:

$$\alpha_{h,a}(T_s) = \varepsilon_{h,a}(T_m), \qquad \alpha_{h,b}(T_s) = \varepsilon_{h,b}(T_m) \tag{3}$$

where

$$T_m = (T_s T_\infty)^{1/2} \tag{4}$$

In our previous work [5], the cooling rate dT_s/dt appearing in Eqs. (1) and (2) was expressed as a power of temperature T_s. In this study, a more reasonable and accurate expression for the cooling curves is used, as described below.

When a specimen of mass m and surface area F is allowed to cool in a vacuum chamber, the power balance for the specimen can be approximately written as

$$-c(T_s)m\frac{dT_s}{dt} = \varepsilon_h(T_s)\,\sigma F T_s^4 \tag{5}$$

Besides, the thermophysical properties c and ε_h of the specimen may be expressed as a function of T_s as follows:

$$\frac{c(T_s)}{\varepsilon_h(T_s)} = A_0(1 + \alpha T_s + \beta T_s^2), \qquad A_0, \alpha, \beta, \qquad \text{constant} \tag{6}$$

Substitution of Eq. (6) into Eq. (5) and integration of the result give

$$t = \frac{h}{T_s^3} + \frac{i}{T_s^2} + \frac{j}{T_s} + k \tag{7}$$

where h, i, j, and k are constants, determined by a least-squares analysis. Equation (7), which is the series in inverse powers of temperature, can be expressed more accurately by T_s versus t curves, which are obtained by the transient calorimetric technique. From Eq. (7), the cooling rate can then be derived as follows:

$$\frac{dT_s}{dt} = \frac{-T_s^4}{3h + 2iT_s + jT_s^2} \tag{8}$$

4. RESULTS AND DISCUSSION

4.1. Specific Heat and Total Hemispherical Emissivity

As described before, the temperatures of specimen II-2 were measured by two thermocouples. From these results and the results for specimen II of concentric-cylinder shape in the previous report [5], it was found that temperature differences between metal A and metal B were very small, by less than 1.5 K at the high temperature and less than 0.2 K at the low temperature. Therefore, the temperatures for specimens II-1 were measured by only one thermocouple to reduce the heat loss Q_t.

Figure 3 shows the cooling curves of specimens I and II-1 for chromel and alumel. To obtain more precise values of the constants in Eq. (7) for the cooling curves, each curve was divided into three parts (770–650, 650–450, and 450–350 K), so that Eq. (7) could be applied to each part with a maximum SEE (standard error of estimate) of 0.7 K. For example, in the temperature range 650–450 K, the constants in Eq. (7) for the chromel specimens were found to be as follows.

Specimen I (chromel):

$$\left.\begin{aligned} h &= 5.0271 \times 10^{11}, & i &= -1.9248 \times 10^{9} \\ j &= 3.1833 \times 10^{6}, & k &= -1.9723 \times 10^{3} \end{aligned}\right\} \tag{9}$$

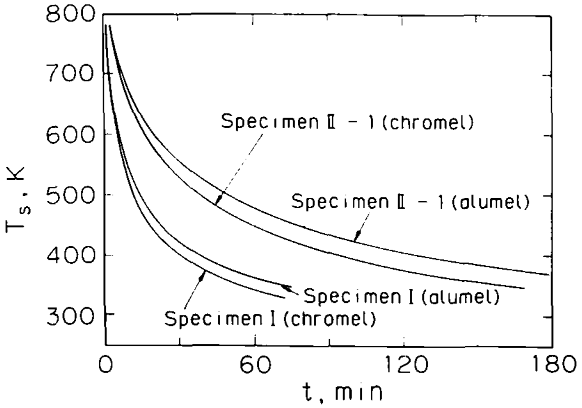

Fig. 3. Cooling curves of specimen temperature versus time for chromel and alumel specimens.

Specimen II-1 (chromel and copper):

$$\left.\begin{aligned} h &= 1.0875\times10^{12}, & i &= -3.1782\times10^{9} \\ j &= 4.4956\times10^{6}, & k &= -2.7324\times10^{3} \end{aligned}\right\} \qquad (10)$$

Here t is time in s and T_s is temperature in K in Eq. (7).

In the calculations of c_a and $\varepsilon_{h,a}$ using Eqs. (1) and (2), the data of Lyusternik [8] and Masuda and Higano [9] were used as the thermophysical properties of the standard reference metal (copper), c_b and $\varepsilon_{h,b}$, respectively. Additionally, the $\varepsilon_{h,b}$ values of copper were given by the following expression, obtained from our data for a disk-shaped specimen [9]:

$$\varepsilon_{h,b(cop)} = 0.0027 + 2.72\times10^{-5}T \qquad (11)$$

The c values obtained from Eq. (1) for the chromel specimens are plotted as a function of temperature T in Fig. 4. In Fig. 4, open circles indicate the values calculated from the data for specimen I (chromel) and specimen II-1 (chromel and copper), and filled circles indicate the values from the data for specimen I (chromel) and specimen II-2 (chromel and copper). The values obtained agree with each other within 7%, although the differences are larger at lower temperatures. Very few c data for chromel have been published thus far. The data of Douglas and Victor [10] for nickel alloy (Ni, 89.1%; Cr, 9.6%; Fe, 0.63%; Si, 0.42%; Zr, 0.12%) are plotted in Fig. 4. The chemical composition of the alloy is somewhat different, but

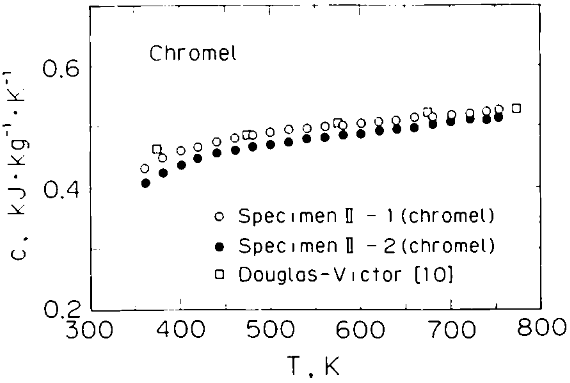

Fig. 4. Specific heat of chromel.

their data agree well with the present results. The ε_h values obtained from Eq. (2) for the chromel specimens are shown in Fig. 5. The results for specimens II-1 and II-2 agree with each other with a maximum difference of 9%, although the differences are larger at lower temperatures again, as in the case of the c values. There are also very few data for the ε_h of chromel. Only the results of Aldao and Löffler [11] using a steady-state method are given in Fig. 5 for comparison. Their data are much larger in comparison with the present results, by about 55% at 600 K. Their data are considered to be of a low accuracy, because their measurements were carried out in a vacuum of 10^{-2} Pa and the results for ε_h of both chromel and alumel were given by the same expression, as a function of T.

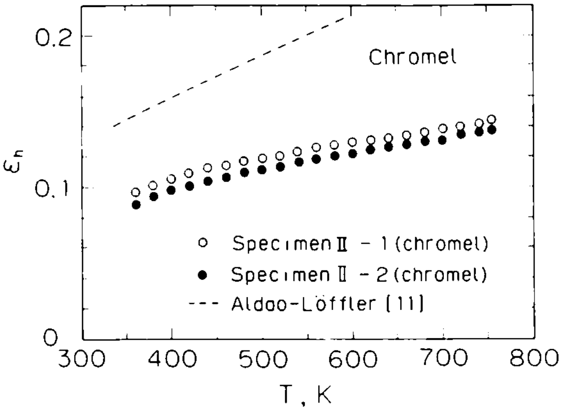

Fig. 5. Total hemispherical emissivity of chromel.

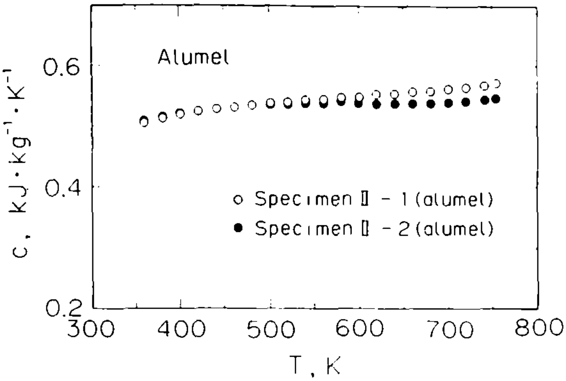

Fig. 6. Specific heat of alumel.

The c values of alumel obtained for specimens II-1 and II-2 are shown in Fig. 6. The increase in c with temperature agrees in trend with the results for chromel. The differences in c values between specimen II-1 and spacimen II-2 are less than 4%. Other data for the specific heat of alumel are unavailable in the literature. The measured values of ε_h of alumel in this work are shown in Fig. 7. The values for specimens II-1 and II-2 agree well with each other to within 4%. The total hemispherical emissivities of alumel are somewhat lower than those of chromel, and the differences become larger at lower temperatures, by about 16% at 360 K. In Fig. 7, ε_h data for alumel of Aldao and Löffler [11] are also indicated. Their results are, once again, much higher than the present results.

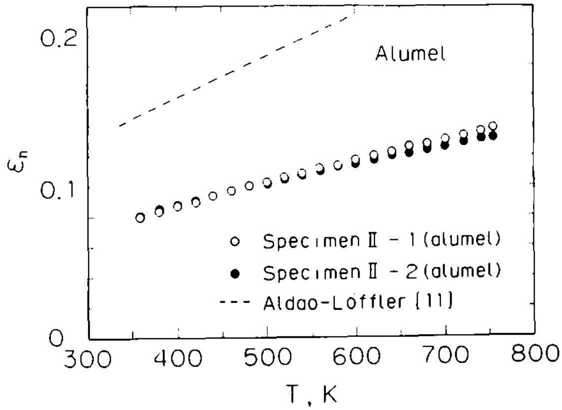

Fig. 7. Total hemispherical emissivity of alumel.

The measured values of c and ε_h for specimens II-1 of chromel and alumel are listed in Table II; their measuring accuracies are presumed to be higher than those of specimens II-2 as described in Section 4.2.

4.2. Estimate of Errors

For the measurements of total emissivity and other properties, error estimates were made previously by Nelson and Bevans [12] for the commonly used calorimetric method and by Cezairliyan et al. [13] for the dynamic, resistive self-heating method in detail. An error analysis is made for the experimental results obtained by the present technique. Uncertainties (total errors) in the measured values of the specific heat and

Table II. Experimental Results on Specific Heat c and Total Hemispherical Emissivity ε_h of Chromel and Alumel

Temperature (K)	Chromel[a] c (kJ·kg^{-1}·K^{-1})	Chromel[a] ε_h	Alumel[b] c (kJ·kg^{-1}·K^{-1})	Alumel[b] ε_h
360	0.431	0.096	0.504	0.077
380	0.447	0.100	0.511	0.080
400	0.458	0.103	0.517	0.083
420	0.466	0.106	0.522	0.087
440	0.473	0.109	0.526	0.090
460	0.479	0.112	0.530	0.094
480	0.484	0.115	0.534	0.097
500	0.488	0.117	0.537	0.100
520	0.492	0.120	0.540	0.103
540	0.495	0.122	0.543	0.106
560	0.499	0.124	0.545	0.109
580	0.502	0.126	0.548	0.112
600	0.504	0.128	0.550	0.115
620	0.507	0.130	0.553	0.118
640	0.510	0.132	0.555	0.121
660	0.512	0.134	0.558	0.124
680	0.515	0.136	0.560	0.127
700	0.518	0.138	0.563	0.130
720	0.521	0.140	0.566	0.133
740	0.525	0.142	0.570	0.136
760	0.528	0.144	0.573	0.139

[a] Values obtained for chromel specimens I and II-1.

[b] Values obtained for alumel specimens I and II-1.

total hemispherical emissivity in this work, $\Delta c_a/c_a$ and $\Delta\varepsilon_{h,a}/\varepsilon_{h,a}$, can be expressed, noting Eqs. (1) and (2), as

$$\frac{\Delta c_a}{c_a}=\left\{\left[\frac{\Delta c_a}{c_a}\right]^2_{\mathrm{phys}}+\left[\frac{\Delta c_a}{c_a}\right]^2_{\mathrm{cool}}+\left[\frac{\Delta c_a}{c_a}\right]^2_{\mathrm{conv}}+\left[\frac{\Delta c_a}{c_a}\right]^2_{\delta T_s}+\left[\frac{\Delta c_a}{c_a}\right]^2_{Q_l}+\left[\frac{\Delta c_a}{c_a}\right]^2_{\delta\varepsilon_{h,a}}\right\}^{1/2} \tag{12}$$

and

$$\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}=\left\{\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{c_a}+\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{\mathrm{cool}}+\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{\mathrm{conv}}+\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{\delta T_s}+\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{Q_l}+\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]^2_{\delta\varepsilon_{h,a}}\right\}^{1/2} \tag{13}$$

Here, the errors due to the heat loss through the residual gas in the vacuum chamber of the experimental apparatus and the hot spot from the terminal box (see 9 in Fig. 2) are neglected, since they are extremely small.

All the terms on the right side of Eqs. (12) and (13) denote error factors contributing to the total error $\Delta c_a/c_a$ or $\Delta\varepsilon_{h,a}/\varepsilon_{h,a}$. First, the term $[\;]_{\mathrm{phys}}$ represents the error attributable to total errors in the values of c_b, $\varepsilon_{h,b}$, and $\alpha_{h,b}$, i.e., in the thermophysical property data of metal B (the standard metal). Next, the terms $[\;]_{\mathrm{cool}}$ represent the error involved in evaluating the cooling rates of the specimens, $dT_{s,\mathrm{I}}/dt$ and $dT_{s,2}/dt$, given by Eq. (8). The terms $[\;]_{\mathrm{conv}}$ represent the error involved in measuring the masses, surface areas, and temperatures of specimens I and II and the surrounding wall temperature. The terms $[\;]_{\delta T_s}$ represent the error caused by assuming that, for compound specimen II, the mean value of temperatures measured by the two thermocouples is the reference temperature of the specimen $T_{s,2}$. The terms $[\;]_{Q_l}$ represent the error involved in assuming values for ε_h of the thermocouple leads in the calculation of Q_l. The terms $[\;]_{\delta\varepsilon_{h,a}}$ represent the error caused by a slight difference in the real $\varepsilon_{h,a}$ values between specimen I and specimen II. Finally, the term $[\Delta\varepsilon_{h,a}/\varepsilon_{h,a}]_{c_a}$ represents the error in $\varepsilon_{h,a}$, which is caused by several errors in the measured value of c_a obtained by an error analysis from Eq. (1). All the above-mentioned errors can be given by a usual error analysis, based on Eqs. (1) and (2), except for the error $[\;]_{\delta\varepsilon_{h,a}}$. The analytical results for several errors of importance are discribed below.

The expressions for errors $[\Delta c_a/c_a]_{\mathrm{phys}}$ and $[\Delta c_a/c_a]_{\mathrm{cool}}$ may be obtained from Eqs. (1) and (2), neglecting terms which are relatively small, e.g., the term $Q_{r,\mathrm{I}}$:

$$\left[\frac{\Delta c_a}{c_a}\right]_{\mathrm{phys}}=\left[\left(\frac{p}{p-1}\frac{\Delta c_b}{c_b}\right)^2+\left(\frac{1}{1-p}\frac{\Delta\varepsilon_{h,b}}{\varepsilon_{h,b}}\right)^2\right]^{1/2} \tag{14}$$

and

$$\left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]_{\rm cool} = \left\{\left[\frac{1}{q-1}\left(\Delta \frac{dT_{\rm s,1}}{dt}\Big/\frac{dT_{\rm s,1}}{dt}\right)\right]^2 + \left[\frac{p-q}{(p-1)(1-q)}\left(\Delta \frac{dT_{\rm s,2}}{dt}\Big/\frac{dT_{\rm s,2}}{dt}\right)\right]^2\right\}^{1/2} \qquad (15)$$

where p and q are factors expressed as

$$p = \frac{-m_{\rm b,2}c_{\rm b}(T_{\rm s})(dT_{\rm s,2}/dt)|_{T_{\rm s}}}{\sigma F_{\rm b,2}[\varepsilon_{\rm h,b}(T_{\rm s})\,T_{\rm s}^4 - \alpha_{\rm h,b}(T_{\rm s})\,T_\infty^4]} \qquad (16)$$

$$q = \frac{-m_{\rm a,2}c_{\rm a}(T_{\rm s})(dT_{\rm s,2}/dt)|_{T_{\rm s}}}{\sigma F_{\rm a,2}[\varepsilon_{\rm h,a}(T_{\rm s})\,T_{\rm s}^4 - \alpha_{\rm h,a}(T_{\rm s})\,T_\infty^4]} \qquad (17)$$

In other words, p and q are shape factors depending on the shape and composition of metals B and A, respectively, for specimen II. In Eq. (14), $\Delta c_{\rm b}$ and $\Delta\varepsilon_{\rm h,b}$ are the total errors involved in $c_{\rm b}$ and $\varepsilon_{\rm h,b}$ of the standard metal B, respectively, which have been estimated to be 1 and 3%, respectively, as described in Ref. 5. The quantities $\Delta(dT_{\rm s,1}/dt)$ and $\Delta(dT_{\rm s,2}/dt)$ in Eq. (15) represent errors in the values of $dT_{\rm s,1}/dt$ and $dT_{\rm s,2}/dt$, respectively, caused by expressing the cooling curve of the specimen by Eq. (7).

The error $[\Delta\varepsilon_{\rm h,a}/\varepsilon_{\rm h,a}]_{c_{\rm a}}$ in Eq. (13) can be expressed as

$$\left[\frac{\Delta\varepsilon_{\rm h,a}}{\varepsilon_{\rm h,a}}\right]_{c_{\rm a}} = \left\{\left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]^2_{\rm phys} + \left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]^2_{\rm cool} + \left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]^2_{\rm conv} + \left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]^2_{\delta T_{\rm s}} + \left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]^2_{Q_{\rm t}}\right\}^{1/2} \qquad (18)$$

The term $[\;]_{\delta\varepsilon_{\rm h,a}}$ should be estimated, considering it as an error in $c_{\rm a}$ or $\varepsilon_{\rm h,a}$, although the quantity $\delta\varepsilon_{\rm h,a}$ mentioned before is assumed to be extremely small. The resultant equations can be written, as discussed in the Appendix, as

$$\left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]_{\delta\varepsilon_{\rm h,a}} = \frac{1}{q-1}\,\frac{\delta\varepsilon_{\rm h,a}}{\varepsilon_{\rm h,a}} \qquad (19)$$

$$\left[\frac{\Delta\varepsilon_{\rm h,a}}{\varepsilon_{\rm h,a}}\right]_{\delta\varepsilon_{\rm h,a}} = \left[\frac{\Delta c_{\rm a}}{c_{\rm a}}\right]_{\delta\varepsilon_{\rm h,a}} \qquad (20)$$

The value $\delta\varepsilon_{\rm h,a}/\varepsilon_{\rm h,a}$ is estimated to be ±1 to ±2% at most in this study, though it is difficult to estimate exactly the value.

The calculated results for the aforementioned main errors are described as follows. Equation (14) indicates that the error $[\Delta c_a/c_a)_{\text{phys}}$ is significantly affected by p (positive value) given by Eq. (16), as well as the total errors, Δc_b and $\Delta\varepsilon_{h,b}$, of the standard metal. The quantity p is determined mainly by the shape of the metal B in the specimen and its thermophysical properties. In Fig. 8, the variation of the error $[\Delta c_a/c_a]_{\text{phys}}$ with p is shown. The error $[\Delta c_a/c_a]_{\text{phys}}$ is found to be infinite at $p = 1$. As the factor p increases in the range $p > 1$, the error decreases and tends to the total error in c_b, while in $p < 1$ the error tends to the total error in $\varepsilon_{h,b}$ as the factor p decreases. Taking account of this, specimen II must be designed with the condition $p \gg 1$, that is, the following conditions as described in the previous paper [4], because the value of $\Delta c_b/c_b$ is usually less than that of $\Delta\varepsilon_{h,b}/\varepsilon_{h,b}$.

$$m_{a,2} \ll m_{b,2} \quad \text{and/or} \quad F_{a,2} \gg F_{b,2} \tag{21}$$

In fact, the values of p were in the range 27 to 42. The results obtained for $\Delta c_b/c_b = 1\,\%$ and $\Delta\varepsilon_{h,b}/\varepsilon_{h,b} = 3\,\%$ and for the range of p in this work are indicated by the dotted area on the solid line in Fig. 8. The $\varepsilon_{h,b}$ values of specimen II, however, may be considered to differ very slightly from the emissivity of the specimen used in the previous paper [9], by 1–2%. Hence, assuming the total error in $\varepsilon_{h,b}$ of the specimens in this work to be 5%, not to be 3%, but keeping $\Delta c_b/c_b = 1\,\%$, the errors $[\Delta c_a/c_a]_{\text{phys}}$ for chromel and alumel specimens were calculated from Eq. (14) once again. Then it is confirmed that both the errors are about 1% and almost equal

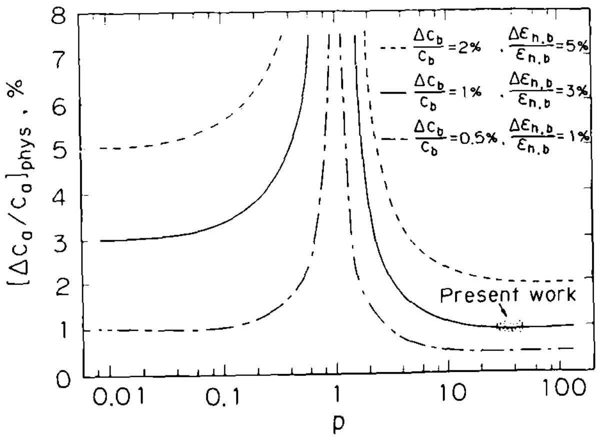

Fig. 8. Variation of error $[\Delta c_a/c_a]_{\text{phys}}$ with the factor p.

to the above value calculated with $\Delta\varepsilon_{h,b}/\varepsilon_{h,b} = 3\%$, because the copper surface in specimen II is relatively small.

Similarly, by inspection of Eq. (15), it may be seen that the error $[\Delta c_a/c_a]_{cool}$ is also affected by the quantities p and q (positive) as well as the errors in the cooling rates of specimens I and II, i.e., $dT_{s,1}/dt$ and $dT_{s,2}/dt$. Figure 9 illustrates the relationship between q and $[\Delta c_a/c_a]_{cool}$, which has been obtained as $\Delta(dT_{s,1}/dt)/(dT_{s,1}/dt) = \Delta(dT_{s,2}/dt)/(dT_{s,2}/dt)$, for chromel and alumel specimens. The error $[\Delta c_a/c_a]_{cool}$ is infinite at $q = 1$, similar to the case of $[\Delta c_a/c_a]_{phys}$ at $p = 1$. And yet, as the factor q departs from unity, the error $[\Delta c_a/c_a]_{cool}$ tends to a constant value depending on p and the cooling rates of specimens I and II. In the design of specimen II, therefore, the quantity of q must also be taken away from unity, similar to that of p for the error $[\Delta c_a/c_a]_{phys}$. However, the factor q has a characteristic contrary to p in quantity, so that a value less than unity may be taken as the quantity of q. As a result, the quantities of q were in the range 0.2 to 0.7 in this work. The errors in $dT_{s,1}/dt$ and $dT_{s,2}/dt$ in Eq. (15) are both less than 0.5%. From these results, the error $[\Delta c_a/c_a]_{cool}$ is estimated to be less than 0.9% for specimens II-1 and to be less than 2.4% for specimens II-2. The error results for the specimens in the present work are shown in Fig. 9 by the dotted area on the solid line for $\Delta(dT_s/dt)/(dT_s/dt) = 0.5$.

The relationship between q and $[\Delta c_a/c_a]_{\delta\varepsilon_{h,a}}$ (in absolute value) is shown in Fig. 10 with $\delta\varepsilon_{h,a}/\varepsilon_{h,a}$ as a parameter. This error is less than 2.7% for specimens II-1 and less than 7.0% for specimens II-2 under the assumption that $\delta\varepsilon_{h,a}/\varepsilon_{h,a}$ is $\pm 2\%$. In the present work, the errors $[\ ]_{\delta\varepsilon_{h,a}}$ are found to be the largest among various errors in c_a, as indicated in Fig. 11.

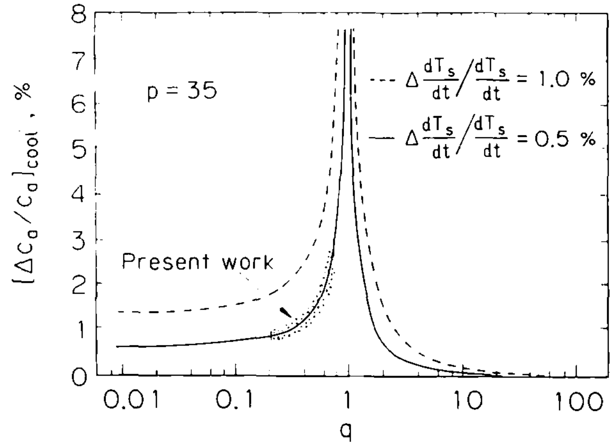

Fig. 9. Variation of error $[\Delta c_a/c_a]_{cool}$ with the factor q.

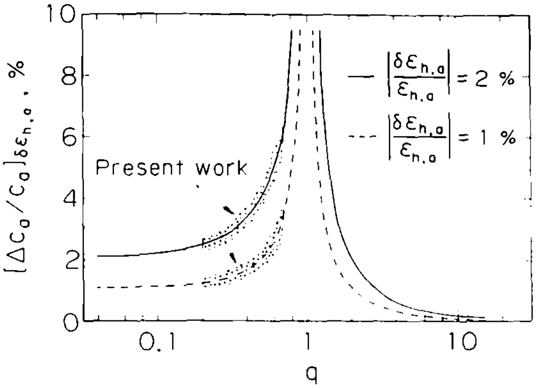

Fig. 10. Variation of error $[\Delta c_a/c_a]_{\delta\varepsilon_{h,a}}$ (absolute value) with the factor q.

For chromel, as an example, the total error and various errors in specific heat and total hemispherical emissivity are shown in Figs. 11 and 12, respectively, for specimen II-1, whose total errors were estimated to be smaller than those of specimen II-2. In these figures, the error $[\ ]_{Q_l}$ in the above calculations was estimated by examining the effect of the ε_h values of the used thermocouple leads on the heat loss Q_l under the assumption that the real values (both iron's and constantan's ε_h) are one and a half times as high as the values of the highly polished surface used in the first calculation in Section 4.1. From Figs. 11 and 12, it can be seen that both errors of $[\Delta c_a/c_a]_{\delta\varepsilon_{h,a}}$ and $[\Delta\varepsilon_{h,a}/\varepsilon_{h,a}]_{\delta\varepsilon_{h,a}}$ are much larger than the other errors in c_a and $\varepsilon_{h,a}$, respectively. Hence, it may be noted that the surface

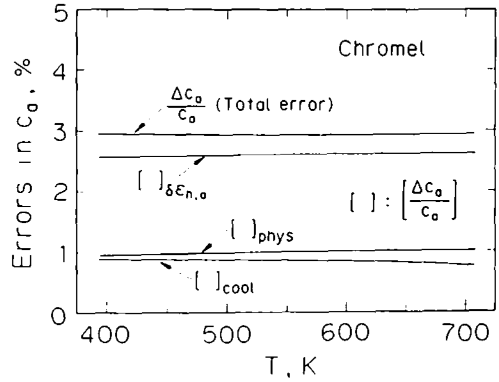

Fig. 11. Total error and various errors in c values of chromel (chromel specimen II-1).

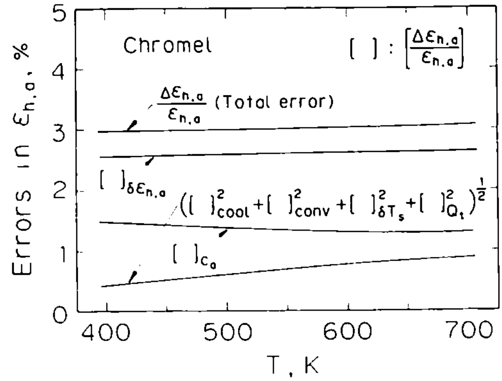

Fig. 12. Total error and various errors in ε_h values of chromel (chromel specimen II-1).

finishing of specimens I and II used in the present technique not only with great care must be made, but also should be the same in order to reduce the total error as well as the errors $[\ ]_{\delta\varepsilon_{h,a}}$. The total error (uncertainty) in the measured values of the specific heat of chromel and alumel was estimated to be 3.1% for specimens II-1 and 7.5% for specimens II-2. Furthermore, the total error in the total hemispherical emissivity of chromel and alumel was also estimated to be 3.1% for specimens II-1 and 7.5% for specimens II-2.

5. CONCLUSIONS

The specific heat and total hemispherical emissivity of chromel and alumel were measured by means of the simultaneous measurement technique developed by Masuda et al. [5]. An expression which accurately predicts the cooling rates of the specimens was developed to obtain more precise values of the thermophysical properties. A detailed error analysis was made for the values measured by the present technique. The total error and the other various errors contributing to the total error in the measured values are discussed. The total error in specific heat and total hemispherical emissivity is estimated to be 3.1% for both chromel and alumel specimens (the well-designed specimens II-1).

APPENDIX: ESTIMATE OF ERRORS CAUSED BY THE DIFFERENCES $\delta\varepsilon_{h,a}$

The errors in c_a and $\varepsilon_{h,a}$, $[\ ]_{\delta\varepsilon_{h,a}}$, caused by the difference $\delta\varepsilon_{h,a}$ in the real emissivity values of metal A between specimen I and specimen II are estimated, though the difference is considered to be very slight. The real emissivities of metal A in specimens I and II are denoted $\varepsilon_{\mathrm{h,a}}$ and $\varepsilon_{\mathrm{h,a}} + \delta\varepsilon_{\mathrm{h,a}}$, respectively. The analysis for $[\ ]_{\delta\varepsilon_{\mathrm{h,a}}}$ is made, neglecting the heat loss Q_{t}, which has a very small value.

First, in the present measurement method, taking no account of the difference $\delta\varepsilon_{\mathrm{h,a}}$, power balance equations can be written for specimens I and II, respectively, as

$$-m_{\mathrm{a,1}} c_{\mathrm{a}}(T_{\mathrm{s,1}}) \left.\frac{dT_{\mathrm{s,1}}}{dt}\right|_{T_{\mathrm{s,1}}} = \varepsilon_{\mathrm{h,a}}(T_{\mathrm{s,1}})\, \sigma F_{\mathrm{a,1}}(T_{\mathrm{s,1}}^4 - T_{\infty}^4) \tag{22}$$

$$\begin{aligned} -[m_{\mathrm{a,2}} c_{\mathrm{a}}(T_{\mathrm{s,2}}) + m_{\mathrm{b,2}} c_{\mathrm{b}}(T_{\mathrm{s,2}})] \left.\frac{dT_{\mathrm{s,2}}}{dt}\right|_{T_{\mathrm{s,2}}} \\ = \sigma[\varepsilon_{\mathrm{h,a}}(T_{\mathrm{s,2}})\, F_{\mathrm{a,2}} + \varepsilon_{\mathrm{h,b}}(T_{\mathrm{s,2}})\, F_{\mathrm{b,2}}](T_{\mathrm{s,2}}^4 - T_{\infty}^4) \end{aligned} \tag{23}$$

The specific heat c_{a} and total hemispherical emissivity $\varepsilon_{\mathrm{h,a}}$ determined simultaneously from Eqs. (22) and (23) are denoted, in particular, by c_{a}' and $\varepsilon_{\mathrm{h,a}}'$, respectively. Next, another power balance equation for specimen II, i.e., an exact equation, taking the difference $\delta\varepsilon_{\mathrm{h,a}}$ into consideration, can be expressed as

$$\begin{aligned} -[m_{\mathrm{a,2}} c_{\mathrm{a}}(T_{\mathrm{s,2}}) + m_{\mathrm{b,2}} c_{\mathrm{b}}(T_{\mathrm{s,2}})] \left.\frac{dT_{\mathrm{s,2}}}{dt}\right|_{T_{\mathrm{s,2}}} \\ = \sigma\{[\varepsilon_{\mathrm{h,a}}(T_{\mathrm{s,2}}) + \delta\varepsilon_{\mathrm{h,a}}]\, F_{\mathrm{a,2}} + \varepsilon_{\mathrm{h,b}}(T_{\mathrm{s,2}})\, F_{\mathrm{b,2}}\}(T_{\mathrm{s,2}}^4 - T_{\infty}^4) \end{aligned} \tag{24}$$

Instead of c_{a}' and $\varepsilon_{\mathrm{h,a}}'$, the exact values of the specific heat and emissivity, c_{a}'' and $\varepsilon_{\mathrm{h,a}}''$, are obtained from Eqs. (22) and (24) as

$$c_{\mathrm{a}}''(T_{\mathrm{s}}) = \frac{m_{\mathrm{b,2}} c_{\mathrm{b}}(T_{\mathrm{s}})(dT_{\mathrm{s,2}}/dt)|_{T_{\mathrm{s}}} + \sigma[\varepsilon_{\mathrm{h,b}}(T_{\mathrm{s}})\, F_{\mathrm{b,2}} + \delta\varepsilon_{\mathrm{h,a}} F_{\mathrm{a,2}}](T_{\mathrm{s}}^4 - T_{\infty}^4)}{m_{\mathrm{a,1}}[(F_{\mathrm{a,2}}/F_{\mathrm{a,1}})(dT_{\mathrm{s,1}}/dt)|_{T_{\mathrm{s}}}] - m_{\mathrm{a,2}}(dT_{\mathrm{s,2}}/dt)|_{T_{\mathrm{s}}}} \tag{25}$$

$$\varepsilon_{\mathrm{h,a}}''(T_{\mathrm{s}}) = \frac{-m_{\mathrm{a,1}} c_{\mathrm{a}}''(T_{\mathrm{s}})(dT_{\mathrm{s,1}}/dt)|_{T_{\mathrm{s}}}}{\sigma F_{\mathrm{a,1}}(T_{\mathrm{s}}^4 - T_{\infty}^4)} \tag{26}$$

From c_{a}' determined from Eqs. (22) and (23) and c_{a}'' given by Eq. (25), the error $[\Delta c_{\mathrm{a}}/c_{\mathrm{a}}]_{\delta\varepsilon_{\mathrm{h,a}}}$ becomes

$$\left[\frac{\Delta c_{\mathrm{a}}}{c_{\mathrm{a}}}\right]_{\delta\varepsilon_{\mathrm{h,a}}} = \frac{c_{\mathrm{a}}'' - c_{\mathrm{a}}'}{c_{\mathrm{a}}'} = \frac{1}{q-1}\,\frac{\delta\varepsilon_{\mathrm{h,a}}}{\varepsilon_{\mathrm{h,a}}} \tag{27}$$

Similarly, the error $[\Delta\varepsilon_{h,a}/\varepsilon_{h,a}]_{\delta\varepsilon_{h,a}}$ is also obtained from Eqs. (22), (23), and (26) as

$$\left[\frac{\Delta\varepsilon_{h,a}}{\varepsilon_{h,a}}\right]_{\delta\varepsilon_{h,a}} = \frac{\varepsilon''_{h,a} - \varepsilon'_{h,a}}{\varepsilon'_{h,a}} = \frac{1}{q-1}\,\frac{\delta\varepsilon_{h,a}}{\varepsilon_{h,a}} \tag{28}$$

In turn, the value of $[\Delta\varepsilon_{h,a}/\varepsilon_{h,a}]_{\delta\varepsilon_{h,a}}$ is equal to that of $[\Delta c_a/c_a]_{\delta\varepsilon_{h,a}}$.

ACKNOWLEDGMENTS

The authors would like to thank Mr. N. Watanabe, Ichinoseki National College of Technology, for his technical assistance. We are also grateful to Miss K. Saito, Institute of Fluid Science, Tohoku University, for her assistance in preparing the manuscript.

NOMENCLATURE

c	Specific heat
D	Diameter
F	Surface area
L	Thickness
h, i, j, k	Constants in Eq. (7)
m	Mass
p	Shape factor [Eq. (16)]
Q_t	Conduction heat loss through thermocouple leads suspending specimen
q	Shape factor [Eq. (17)]
T	Temperature
T_s	Specimen temperature
T_∞	Surrounding wall temperature
t	Time
α_h	Total hemispherical absorptivity
Δc_a	Total error in the measured c_a value
Δc_b	Total error in the utilized c_b data
$\Delta\varepsilon_{h,a}$	Total error in the measured $\varepsilon_{h,a}$ value
$\Delta\varepsilon_{h,b}$	Total error in the utilized $\varepsilon_{h,b}$ data
$\delta\varepsilon_{h,a}$	Difference in the real $\varepsilon_{h,a}$ values between specimen I and specimen II
ε_h	Total hemispherical emissivity
σ	Stefan–Boltzmann constant
$[\ ]_{c_a}$	Error attributable to several errors in the measured c_a value [Eq. (18)]

$[\ ]_{\mathrm{conv}}$	Error involved in measuring the masses, surface areas, and temperatures of specimens I and II and the surrounding temperature
$[\ ]_{\mathrm{cool}}$	Error involved in evaluating the cooling rates of the specimens, $dT_{s,1}/dt$ and $dT_{s,2}/dt$ [e.g., Eq. (15)]
$[\ ]_{\mathrm{phys}}$	Error attributable to the total errors in the utilized c_{b} data and $\varepsilon_{\mathrm{h,b}}$ data [e.g., Eq. (14)]
$[\ ]_{Q_l}$	Error involved in evaluating the heat loss Q_l
$[\ ]_{\delta T_s}$	Error attributable to the temperature difference between metal A and metal B in specimen II
$[\ ]_{\delta\varepsilon_{\mathrm{h,a}}}$	Error attributable to the difference in the $\varepsilon_{\mathrm{h,a}}$ values between specimen I and specimen II [Eqs. (19) and (20)]

Subscripts

1	Specimen I
2	Specimen II
a	Metal A to be measured
b	Standard reference metal B
cop	Copper

REFERENCES

1. A. Cezairliyan, *Int. J. Thermophys.* **5**:177 (1984).
2. A. Cezairliyan, *Proc. 2nd Asian Thermophys. Prop. Conf., Sapporo*, N. Seki and B. X. Wang, eds. (1989), pp. 1–7.
3. R. E. Taylor, *High Temp.-High Press.* **13**:9 (1981).
4. I. Takahashi and A. Sugawara, *Trans. JSME Ser. B* **56**:1424 (1990).
5. H. Masuda, S. Sasaki, M. Higano, and H. Sasaki, *Exp. Therm. Fluid Sci.* **4**:218 (1991).
6. H. Masuda and M. Higano, *ASME J. Heat Transfer* **110**:166 (1988).
7. E. R. G. Eckert, *Forsch. Geb. Ingenieurw.* 7:265 (1936).
8. V. E. Lyusternik, *Prib. Tekh. Eksp.* 4:127 (1959); cited by Y. S. Touloukian and E. H. Buyco, eds., *Thermophysical Properties of Matter, Vol. 4* (IFI/Plenum, New York, 1970), pp. 52, 55.
9. H. Masuda and M. Higano, *J. Opt. Soc. Am. A* **2**:1877 (1985).
10. T. B. Douglas and A. C. Victor, *J. Res. Natl. Bur. Stand.* **65C**:65 (1961).
11. C. M. Aldao and D. G. Löffler, *J. Vac. Sci. Technol.* **A2**:1601 (1984).
12. K. E. Nelson and J. T. Bevans, *Measurement of Thermal Radiation Properties of Solids*, J. C. Richmond, ed., NASA SP-31 (1963), pp. 55–65.
13. A. Cezairliyan, M. S. Morse, H. A. Berman, and C. W. Beckett, *J. Res. Natl. Bur. Stand.* **74A**:65 (1970).